DYNAMICS OF FLAT PLATE COMPLETE MOTION IN AIR FLOW WITH VARIABLE VELOCITY

Marina Cerpinska, Janis Viba, Ivo Vaicis
Riga Technical University, Latvia
marina.cerpinska@rtu.lv, janis.viba@rtu.lv, ivo.vaicis@rtu.lv

Abstract. The research presents the mathematical model of non-stationary movement of an object in the air, considering that the wind flow might not be constant, as in the example of the vertical wind tunnel. The air flow rate might change its direction and magnitude over time, having variable velocity – a problem not solvable using conventional methods of aerodynamics, drag and lift force. Instead, the air flow around the object is divided into pressure and suction zones, and the angle of attack is variable. A thin flat plate is chosen as the moving object to allow the object not only translational movement, but also one rotational degree of freedom. Differential equations corresponding to a system with three degrees of freedom are presented in the article. Differential-integral equations are obtained and solved numerically using MathCAD software and explicit Euler step integration method. The developed method of motion analysis has been used for solving specific tasks: 1) vertical free fall of the plate in variable wind flow; 2) oscillations of the plate around a stationary base if additional elastic forces of the suspension reaction are applied. In addition, the case of a flapping wing structure in which energy is extracted from the air flow is considered. The obtained results can be used in practice to reduce or increase the interaction of other similar surface objects with air, for example, for sports equipment in competitions and everyday vehicles, as well as for obtaining energy from variable air flow in wind generators.

Keywords: air interaction, vibrations, variable flow, wind generator.

Introduction

Living organisms and structures are exposed to the wind and variable air flow. As a result, many living organisms developed ability of using air to their advantage. Flying is an excellent example of energy harvesting from the air. Nature granted living organisms with additional capabilities to fly better, and details of fly, feathers and wing structures are still fascinating researchers in bio-inspired robotics area and biomimetics. For example, during the soaring motion of many birds, their wingtip feathers break up the airflow from below, and the feathers are curved upward to create arrays of small winglets, which inspired the specific design of jet planes [1]. Various lift-enhancement mechanisms of flying species are presented in the literature, including leading-edge vortex, rapid pitch-up, wake capture, and clap-and-fling [2]. The flying capability of a bumblebee confused researchers for decades, until it was concluded that the secret of insects' ability to fly is a high speed of moving their wings, moreover, insects' wings act as a flexible membrane [3]. The idea of energy harvesting from flexible structures is discussed in energy and construction research in the latest research [4]. This paper focuses on energy accumulation from gravity and elasticity force.

Unpowered flight, gliding and soaring were researched since the late 15th century by Leonardo da Vinci, and nowadays people learned well to fly artificially with assistive devices (e.g. for paragliding, kiteboarding), or using additional energy supply from a possible energy source (e.g. drones and helicopters). An incomprehensible number of studies has been devoted to the process of interaction of moving objects with air [5]. However, the air flow has an infinite number of particles, which interact differently with a moving object. It is a complicated analysis and has no analytical solution.

The theory of aerodynamics is based on the Newtons' laws and Bernoulli's principle, which are simplified into an integral form. Scientists rationally use the results of experiments in wind tunnels and adjust the parameters in the laws of nature. Computer technology, CFD analysis brought significant progress into the studies, allowing verification on whether the proposed theories are correct. In this work, we further explore the possibility started by previous research [6] of studying the interaction of the air flow with a relatively simple object - a flat plate, without using the concepts of drag and lift forces or their descriptive coefficients. The idea of combining drag and lift forces was expressed as early as in 1995 [7] for design of a plane, because in those times limited computational power forced engineers to take assumptions to ease calculations, but even nowadays, when computational power increased dramatically, the idea of combined aerodynamic forces is discussed in the context of design of high-speed rotating equipment, like wind power generators [6], hydropower turbines [8] and structural applications where development of a control system is considered [9]. The air environment of infinite

DOI: 10.22616/ERDev.2022.21.TF119

degrees of freedom is localized in small areas around the object, determining the magnitudes of the interaction in a differential-integral form. The advantages of this method are that in the case of different movements it is possible to analyse the interactions, optimize the parameters and control of the system and flow, to synthesize new interaction systems. The proposed method is illustrated with calculation examples in this work.

Methods

For the following examples the laws of classical mechanics are used, including:

1. the differential equation of motion of the center of mass of a plate;
2. the relation between the change of the main moment of the plate movement in the differential form;
3. the relationship between the change in the amount of motion in the differential form of the air particles on the boundary layer of the plate.

The theoretical methodology of describing the air flow and object interaction, dividing the area into the pressure and suction zone is described in previous work [10], and brings the relation ($1 \pm C$) into equations (1)-(3). The method proposed is verified with CFD analysis in the paper [10].

In this paper we focus on numerical simulations of differential equations, using the explicit Euler step integration method in MathCAD 14.

Motion of a thin plate in a variable air flow

The model of the thin flat plate is shown in Fig.1. In the fixed reference system XOY, the plane motion of a thin plate is described by two coordinates *x*, *y* of the centre of mass *C* and one angle of rotation φ:

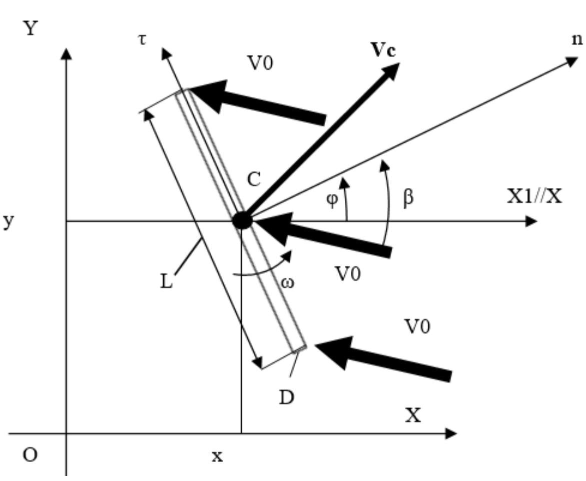

Fig. 1. **Model of thin plate motion**

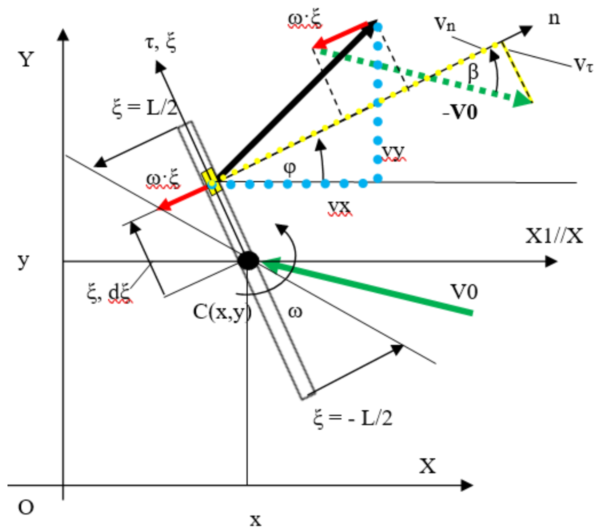

Fig. 2. **Graphs of air interaction components**

According to the description of air interactions of the method developed by the authors of this paper, applying the principle of superposition, which was previously proved to be useful for studies of structures like bridges [11], the forces and the moment can be determined as follows (1)-(3):

$$N = -(1 \pm C) \cdot \frac{1}{2} \cdot \rho \cdot B \cdot L \int_{-\xi}^{\xi} (v_n)^2 \cdot d\xi \cdot \operatorname{sign}(v_n), \tag{1}$$

$$T = -(1 \pm C) \cdot \frac{1}{2} \cdot \rho \cdot B \cdot D \cdot (v_\tau)^2 \cdot \operatorname{sign}(v_\tau), \tag{2}$$

$$M_C = -(1 \pm C) \cdot \frac{1}{2} \cdot \rho \cdot B \cdot L \int_{-\xi}^{\xi} (v_n)^2 \cdot \xi \cdot d\xi \cdot \operatorname{sign}(v_n), \tag{3}$$

where N – force resulting from velocity component V_n, N;
T – force resulting from velocity component V_τ, N;
M_C – moment component resulting from vector V_0, Nm;
C – air flow and object interaction constant, which depends on the shape of the object and surface roughness of the object, unitless;
$(1 + C)$ – relation resulting from dividing air interaction into the pressure and suction zone, where coefficient C stands for the pressure zone, and value 1 results from trigonometric relations of flow and object iterations, see [10];
ρ – density of media, $kg \cdot m^{-3}$;
L – length of plate, m;
B – width of a plate, m;
D – thickness of a plate, m;
ξ – half of the length L of a plate, m;
$v_n(\xi)$ – normal velocity at the local point ξ, $m \cdot s^{-1}$, which considers the wind velocity, see eq. (4);
$v_\tau(\xi)$ – tangential velocity at the local point ξ, $m \cdot s^{-1}$, which considers the wind velocity, eq. (5);
sign(…) – mathematical function that extracts the sign of a real number, thus the function is positive, if the velocity is positive and negative, if the velocity is negative.

Velocity equations are further explained in equations (4) and (5):

$$v_n = V_0 \cdot \cos(\beta) - \dot{x} \cdot \cos(\varphi) - \dot{y} \cdot \sin(\varphi) - \dot{\varphi} \cdot \xi \cdot \mathrm{sign}(\dot{\varphi}), \tag{4}$$

$$v_\tau = V_0 \cdot \sin(\beta) + \dot{x} \cdot \sin(\varphi) - \dot{y} \cdot \cos(\varphi), \tag{5}$$

where V_0 – velocity of the wind, $m \cdot s^{-1}$;
β – angle between V_0 and translation velocity component v_n (Fig. 2.), rad;
φ – angle between x axis and velocity vector n (Fig. 2.), rad;
$\dot{x}$, $\dot{y}$ – translational velocity components v_x, v_y in Fig.2., $m \cdot s^{-1}$.

Using air interaction relationships (1)-(3), the plate center of mass motion theorem and moment of momentum changes theorem, we obtain the following differential equations of motion (6)-(8):

$$m \cdot \ddot{x} = N \cdot \cos(\varphi) - T \cdot \sin(\varphi) + F_x^{(\alpha+c)}, \tag{6}$$

$$m \cdot \ddot{y} = N \cdot \sin(\varphi) + T \cdot \cos(\varphi) + F_y^{(\alpha+c)}, \tag{7}$$

$$J_C \cdot \ddot{\varphi} = M_C + M_C^{(\alpha+c)}, \tag{8}$$

where m – mass of the object, kg;
J_C – moment of inertia for the respective mass, $kg \cdot m^{-2}$;
$\ddot{x}$, $\ddot{y}$, $\ddot{\varphi}$ – components of centre mass acceleration and angular acceleration, $m \cdot s^{-2}$, $rad \cdot s^{-2}$;
N – force resulting from velocity component V_n, N;
T – force resulting from velocity component V_τ, N;
M_C – moment component resulting from vector V_0, Nm;
φ – angle between x axis and velocity vector n (Fig. 2.), rad;
$F_x^{(\alpha + c)}$, $F_y^{(\alpha + c)}$ – components of the active forces, N;
$M_C^{(\alpha + c)}$ – component of the constraint moment, Nm.

The resulting equations are of complex differential form. They need to be solved numerically. However, the main advantage of these equations is that it is possible to describe the effect of various parameters on the non-stationary motion of the plate without using additional experimental studies of drag and lift coefficients. To demonstrate this advantage, in the following section specific examples of analysis and synthesis tasks are provided that can be solved with the proposed method.

Results and discussion

Plate vertical fall modeling in vertical sports wind tunnel

The simple case of plate movement is a vertical free fall (Fig.3). It becomes complicated when one considers variable speed of the wind and adds to the equation the drag force the wind exerts on it. The detailed review on the problem is given in [12]. A model with one degree of freedom with respect to the vertical coordinate $\boldsymbol{y}$ (eq. 7) is presented below in Fig.3.

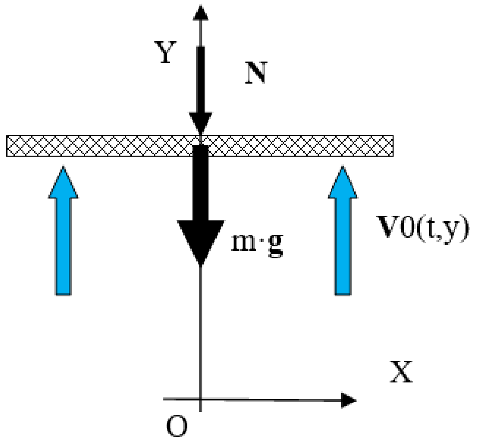

Fig. 3. **Plate in wind tunnel**

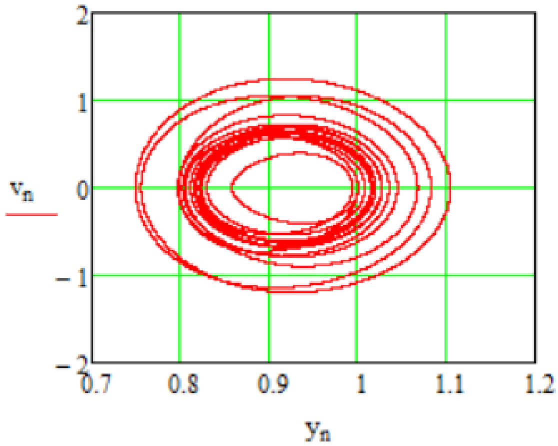

Fig. 4. **Plate centre mass motion in phase plane**

Differential relations (6)-(8) can be provided independently for specific interactions and limits. Let us consider the example of a vertical wind tunnel, where the airflow velocity is vertical with velocity $V0$ (t, y) as a function of time t and vertical coordinate y. Then the motion differential equation will be as follows (9):

$$\begin{cases} V_0(t,y) = 30[1 - 0.1 \cdot \sin(6.283 \cdot t)] \cdot (5 - 4 \cdot y) \\ m \cdot \ddot{y} = (1 \pm C) \cdot \dfrac{1}{2} \cdot \rho \cdot B \cdot L \cdot (V_0(t,y) - \dot{y})^2 \cdot \mathrm{sign}\,(V_0(t,y) - \dot{y}) - m \cdot g \end{cases}, \tag{9}$$

where $V_0(t,y)$ – flow velocity as time t and displacement y function, m·s^{-1};
m – mass, 5 kg;
ρ – density of media, in this example air, 1.25 kg·m^{-3}
C – constant, chosen to be 1.17;
$B{\cdot}L$ – dimensions of the plate, chosen to be B = 0.2 m; L = 1 m;
g – free fall acceleration, 9.8 m·s^{-2}.

The second equation of system (9) is nonlinear, the analysis of it requires additional research and is not limited in scope here. However, an illustrative analysis task, such as a vertical sports wind tunnel, is given in Fig. 4. In this example, the air velocity V_0 is analyzed as a change in harmonic function, with an additional linear y coordinate effect. If one would like to discuss whether formula (9) is not in disagreement with conventional drag and lift equations, then it must be noted that lift depends on the angle of attack and is zero for the angle of attack equal to zero degrees and 90 degrees [13], which is a case of horizontal flat plate in vertical air flow.

Pendulum model of energy extraction from air flow

Imagine the mass of a pendulum, lifted. It accumulates gravitational energy and can transform it to work when released. The greater the mass and height, the greater the potential energy. If initial artificial work done to provide lifting is less than the resulting work (e.g. natural wind is a good example, while the vertical wind tunnel requires deeper feasibility study), the system generates energy.

If we further allow a flat plate to be elastic, then we can consider the energy stored in this beam when compressed like leaf spring metal sheets, as shown in Fig. 5. The stored energy can be calculated using the Euler-Bernoulli beam theory and is descried in other authors works [14]. The motion of the plate oscillations around the horizontal *O* axis (Fig.5) in the horizontal air flow is described. This system has one degree of freedom for rotation, therefore differential equations (6)-(8) are transformed into (10):

$$J_0 \cdot \ddot{\varphi} = (1 \pm C) \cdot \rho \cdot B \cdot \left[\frac{L^2 \cdot V_0(t,\varphi,\dot{\varphi})^2 \cdot (\cos(\varphi))^2}{2} - \frac{2 \cdot L^3 \cdot V_0(t,\varphi,\dot{\varphi}) \cdot \cos(\varphi)}{3} + \frac{L^4 \cdot \dot{\varphi}^2}{4} \right] - \\ - m \cdot g \cdot \frac{L}{2} \cdot \sin(\varphi) - M_e \tag{10}$$

where J_0 – moment inertia of the plate around the horizontal axis, kg·m^{-2};
$\ddot{\varphi}$ – components of the centre mass angular acceleration, rad·s^{-2};
$V_0(t,\varphi,\dot{\varphi})$ – flow velocity, depending on time, angle and angular velocity, in the calculated example it is V0 = 10·(1 + 0.7·sign($\dot{\varphi}$)), m·s^{-1};
M_e – elastic force and moment of the energy generator for a rotating system, in the calculated example it is $M_e = -200 \cdot \varphi - 4 \cdot \dot{\varphi}$, Nm.

It should be noted here that again the differential equation (10) is strongly nonlinear and is not discussed here for analysis. However, for illustration, an example of a controllable system with one boundary cycle, without bifurcations, is shown in Fig. 6.

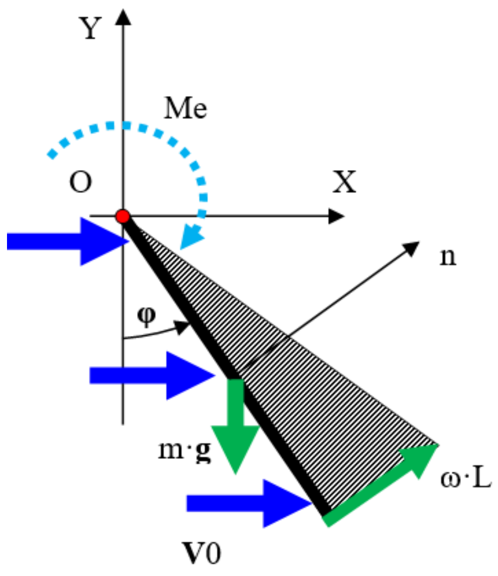

Fig.5. **Oscillating plate**

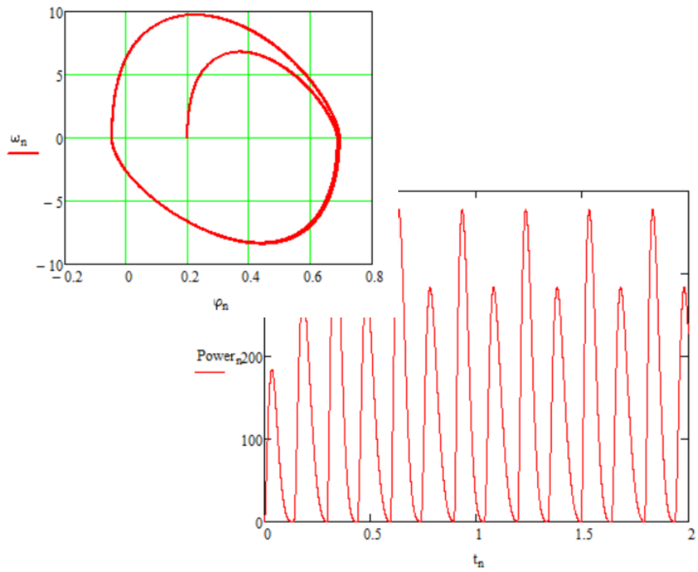

Fig.6. **Motion in phase and power graph**

In this study the principle of superposition is used, so a non-stationary flow interaction is divided into the pressure zone before the object and in the suction area behind the object. Applying Newtons' laws and Bernoulli's principle on the change of the amount of motion of the mechanical system in the differential form of air, which interacts with the plate, the differential equations of the motion of the plate were obtained, which can be used in controlled systems for motion analysis, optimization and synthesis.

Conclusions

1. In this work an approximate method of analysis of the motion of air and simpler objects is proposed, which is based on differential equations in a transparent analytical form.
2. The obtained differential equations, which combine drag and lift concepts into a united coefficient, allow solving object analysis and optimization tasks, as well as control tasks, which arise when energy extraction is considered.
3. The method allowed predicting the object motion in a vertical wind tunnel.
4. The case of plate vertical free fall considering variable speed of the wind was solved with the proposed method. The drag force the wind exerted on it was added to the equation.

Author contributions

M. Cerpinska and J. Viba: writing, review and editing; I. Vaicis: conceptualization, software. All authors have read and agreed to the published version of the manuscript.

References

[1] Prescott T.J., Lepora N.F., Verschure P.F.M.J., Szollosy M., Living Machines. A Handbook of Research in Biomimetic and Biohybrid Systems. Oxford University Press, 2018.

[2] Shyy W., Lian Y., Tang J., Viieru D., Liu H. Aerodynamics of Low Reynolds Number Flyers. Cambridge University Press, 2007.

[3] Truong H., Engels T., Kolomenskiy D., Schneider K. Influence of wing flexibility on the aerodynamic performance of a tethered flapping bumblebee. Theoretical and Applied Mechanics Letters, vol. 10 (6), 2020, pp. 382-389. DOI: 10.1016/j.taml.2020.01.056

[4] Collins I., Hossain M., Dettmer W., Masters I. Flexible membrane structures for wave energy harvesting: A review of the developments, materials and computational modelling approaches. Renewable and Sustainable Energy Reviews, vol. 151 (111478), 2021. DOI: 10.1016/j.rser.2021.111478

[5] Gretchikhin L. Air Pressure Force Acting on a Moving Plate. Scientific Technical Review, vol. 64 (2), 2014, pp.27-33.

[6] Penga Z., Chen B., Qin F.G.F., Jiang R., Tao Q.H.S., Xiao H., Yang Y.C.M. Numerical simulation of aerodynamic performance of an airfoil combined lift and drag. Energy Procedia, vol. 158, 2019, pp. 6282-6287. DOI:10.1016/j.egypro.2019.01.446

[7] Strojnik A., Low power laminar aircraft structures. 7th printing. Arizona State university, 1995.

[8] Basumatary M., Biswas A., Misra R.D. Experimental verification of improved performance of Savonius turbine with a combined lift and drag based blade profile for ultra-low head river application. Sustainable Energy Technologies and Assessments, vol. 44 (100999), 2021. DOI:10.1016/j.seta.2021.100999

[9] Zheng C., Wang Z., Zhang J., Wua Y., Jin Z., Chen Y. Effect of the combined aerodynamic control on the amplitude characteristics of wind loads on a tall building. Engineering Structures, vol. 245 (15), 2021. DOI:10.1016/j.engstruct.2021.112967

[10] Tipans I., Viba J., Irbe M., Vutukuru S.K. Analysis of non-stationary flow interaction with simple form objects. Agronomy Research Journal 2019:17(S1):1227-1234. DOI: 10.15159/AR.19.083.

[11] Siedziako B., Øiseth O. Superposition principle in bridge aerodynamics: Modelling of self-excited forces for bridge decks in random vibrations. Engineering Structures, vol. 179, 2019, pp. 52-65. DOI: 10.1016/j.engstruct.2018.10.072

[12] Souza P.V.S., Girardi D., de Oliveira P.M.C. Drag force in wind tunnels: A new method. Physica A: Statistical Mechanics and its Applications, vol. 467, 2017, pp. 120-128. DOI: 10.1016/j.physa.2016.09.049

[13] Anderson R.J. Introduction to mechanical vibrations. First edition. UK: Wiley, 2020. 268 p.

[14] Truong H., Engels T., Kolomenskiy D., Schneider K. A mass-spring fluid-structure interaction solver: Application to flexible revolving wings. Computers & Fluids. Vol. 200 (104426), 2020. DOI:10.1016/j.compfluid.2020.104426.